Karlsruhe Institute of Technology

Static Program Checking

Bounded Verification – Other Ideas

Automated Software Analysis Group, Institute of Theoretical Informatics

Juniorprof. Dr. Mana Taghdiri

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Incremental bounded verification

- Problems of bounded verification:
  - The formulas generated for non-trivial programs are complex
  - They often choke the solver
    - When the solver times out, there’s no feedback (on coverage of the analysis or likelihood of correctness)
- Solution:
  - Divide the program into several sub-programs
  - Check the property in each sub-program
    - Hopefully each sub-program generates a smaller sub-formula
- Approach:
  - Can partition the program based on control flow
  - Or based on data flow (variable definitions)

Program partitioning

- Proposed for bounded executions
  - Loops are unrolled
- Partition the set of program paths to multiple subsets:

$$path(Proc) = \bigcup_{i=1}^{n} path(Sub_i)$$

- Then, instead of checking $Pre \wedge translate(Proc) \wedge \neg Post$
- We can check

$$\{Pre \wedge translate(Sub_1) \wedge \neg Post\} \wedge .... \wedge \{Pre \wedge translate(Sub_n) \wedge \neg Post\}$$

Partitioning based on control flow

- Splitting algorithm is based on vertices of the computation graph
- Given a vertex, construct two subgraphs
  - Go-through subgraph
  - Bypass subgraph
- Rationale
  - Number of branches is a heuristic metric for complexity
  - Pick a vertex that results in subgraphs with fewer branches
- The splitting can be done recursively as much as desired

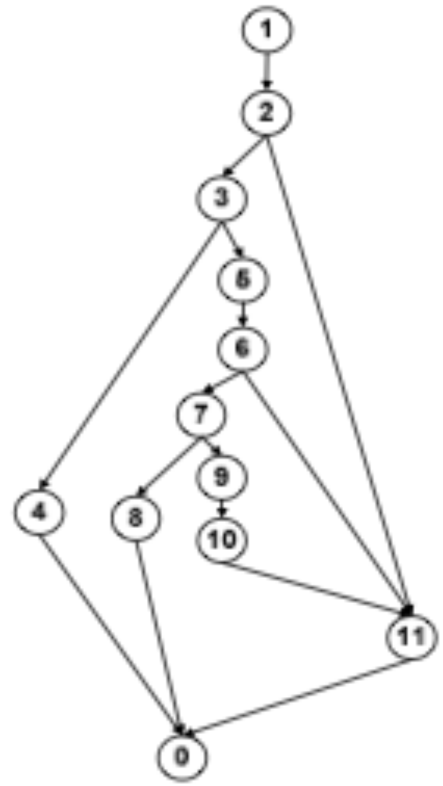
Example

```
class IntList {
   Entry header;
   class Entry {
      int value;
      Entry next;
   };

   boolean contains(int key) {
       Entry e = this.header;
       while (e != null) {
           if (e.value == key)
              return true;
           e = e.next;
       }
       return false;
  }
}
```

Example after two loop unrollings

```
  public boolean
  constains(int key)
   {
1 : Entry e = this.header;
2 : if (e != null){
3 :  if (e.value == key){
4 :   return true;
     }
5 :  e = e.next;
6 :  if (e != null){
7 :   if (e.value == key){
8 :    return true;
      }
9 :   e = e.next;
     }
10:  assume(e == null);
    }
11: return false;
0 :}
```

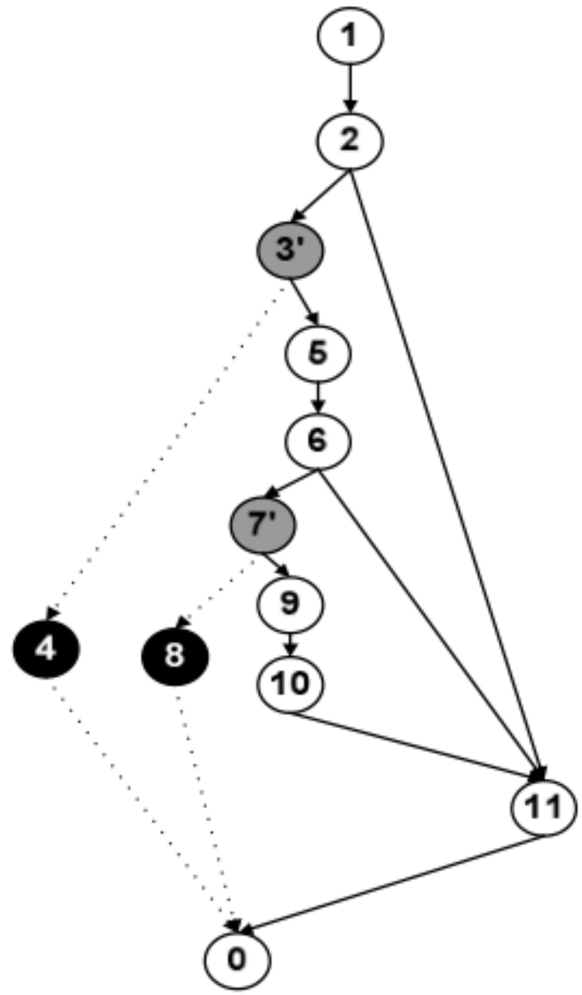


Partition based on node 11

```
  public boolean
  go-through(int key)
   {
1 : Entry e = this.header;
2 : if (e != null){
3' : assume !(e.value==key);
4 :

5 :  e = e.next;
6 :  if (e != null){
7' :  assume!(e.value==key);
8 :

9 :   e = e.next;
    }
10:  assume(e == null);
   }
11: return false;
0 :}
```



Gray: branch converted to assume
Black: removed statements

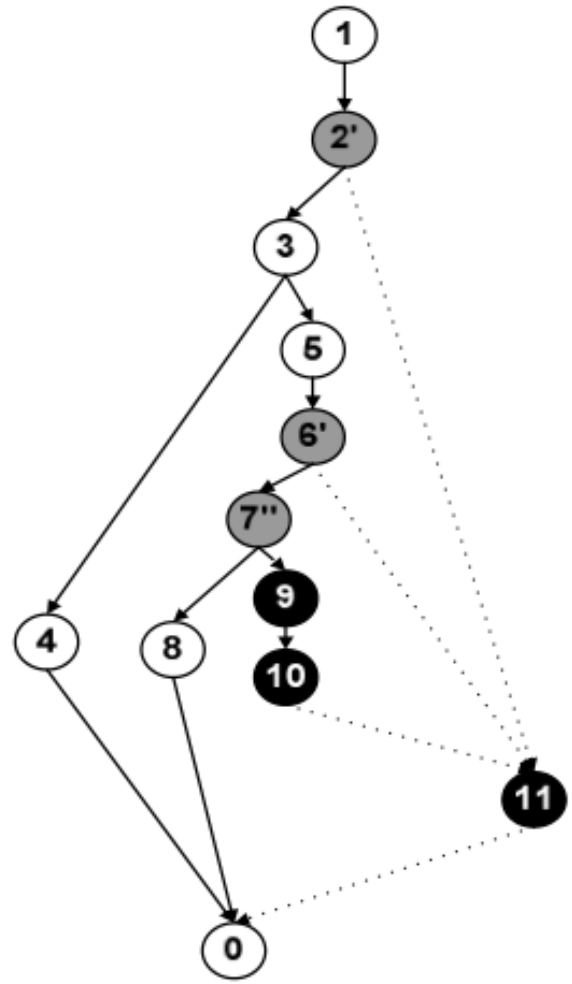
Partition based on node 11

```
  public boolean
  bypass(int key)
  {
1 : Entry e = this.header;
2′ : assume(e != null);
3 : if (e.value == key){
4 :   return true;
    }
5 : e = e.next;
6′ : assume (e != null);
7″: assume(e.value == key);
8 : return true;

9 :

10:

11:
0 :}
```



Gray: branch converted to assume
Black: removed statements

Data flow partitioning

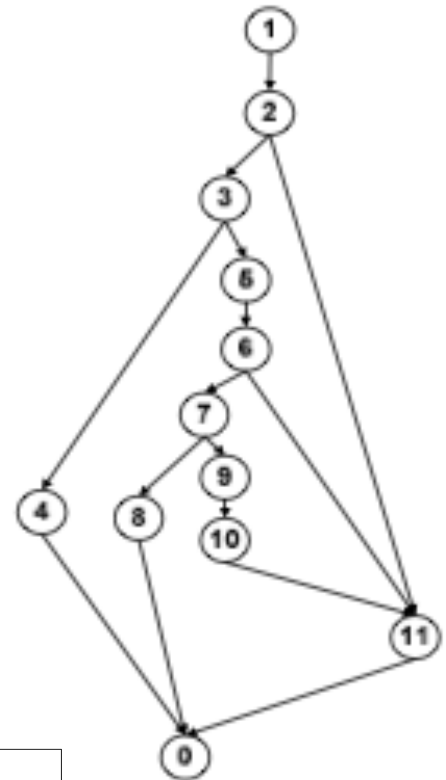
- Control-flow partitioning
  - Is limited to syntactical structure of program
  - Doesn’t exploit program semantics

- Data-flow partitioning is based on variable-definitions
  - Fewer definitions of a variable result in fewer intermediate variables
  - Thus, reduces the number of frame conditions encoding data flow
  - Thus, there are fewer variables in the resulting formula
  - (uses a Jalloy-like translation of code)

- Pick a variable in the computation graph
  - Split the graph into multiple subgraphs s.t. each subgraph has at most one definition for that variable, that can reach the exit statement
  - The definition of this variable is different in each subgraph

Example after two loop unrollings

```
  public boolean
  constains(int key)
   {
1 : Entry e = this.header;
2 : if (e != null){
3 :  if (e.value == key){
4 :   return true;
     }
5 :  e = e.next;
6 :  if (e != null){
7 :   if (e.value == key){
8 :    return true;
      }
9 :    e = e.next;
     }
10:  assume(e == null);
    }
11: return false;
0  :}
```



Definition set of this = {}
Definition set of key = {}
Definition set of return = {4, 8, 11}
Definition set of e = {1, 5, 9}
All of these definitions can reach the exit statement

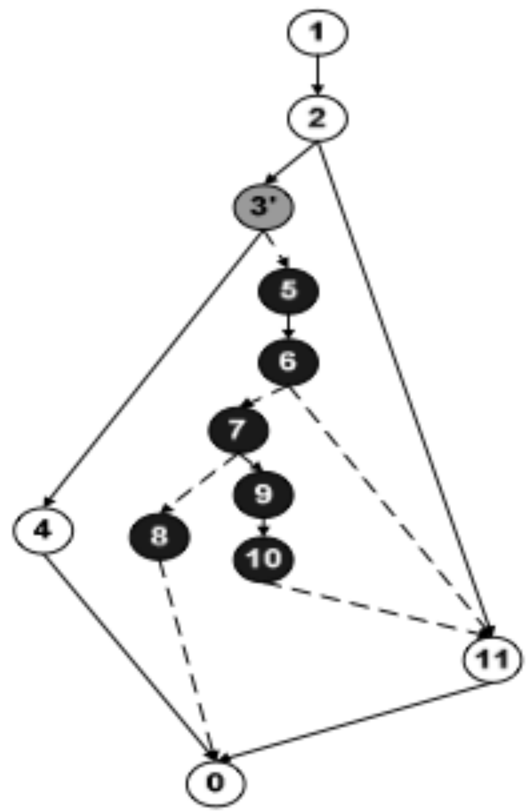
Splitting based on “e”

```
   public boolean
   sub1(int key)
    {
1 : Entry e = this.header;
2 : if (e != null){
3': assume (e.value==key)
4 :   return true;

5 :
6 :
7 :
8 :

9 :

10:
     }
11: return false;
0 :}
```



Now we have exactly one definition of e (line 1)
(doesn’t include 5 or 9)
Set the branch conditions s.t. unwanted nodes are not visited

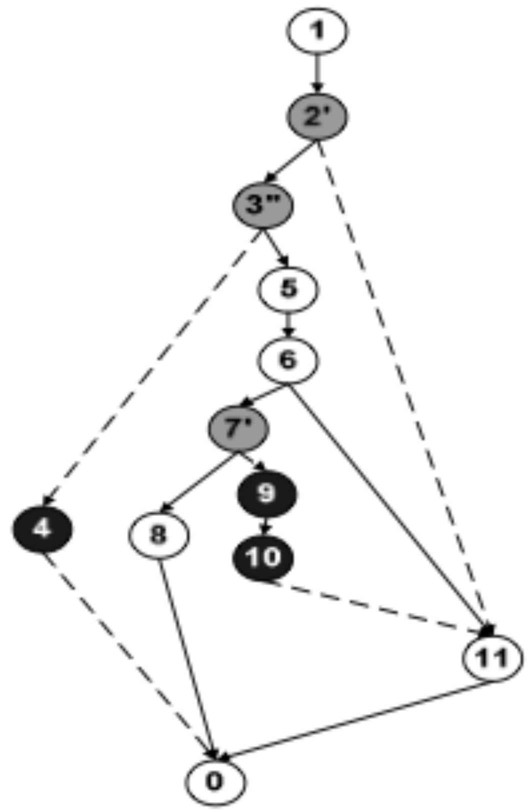
Splitting based on “e”

```
   public boolean
   sub2(int key)
    {
1 : Entry e = this.header;
2': assume (e != null);
3": assume !(e.value==key);
4 :

5 :  e = e.next;
6 :  if (e != null){
7':   assume(e.value==key);
8 :   return true
     }
9 :

10:

11: return false;
0 :}
```



Again exactly one definition of e reaches exit (line 5)
(1 or 9 can’t reach the exit)

Splitting based on “e”

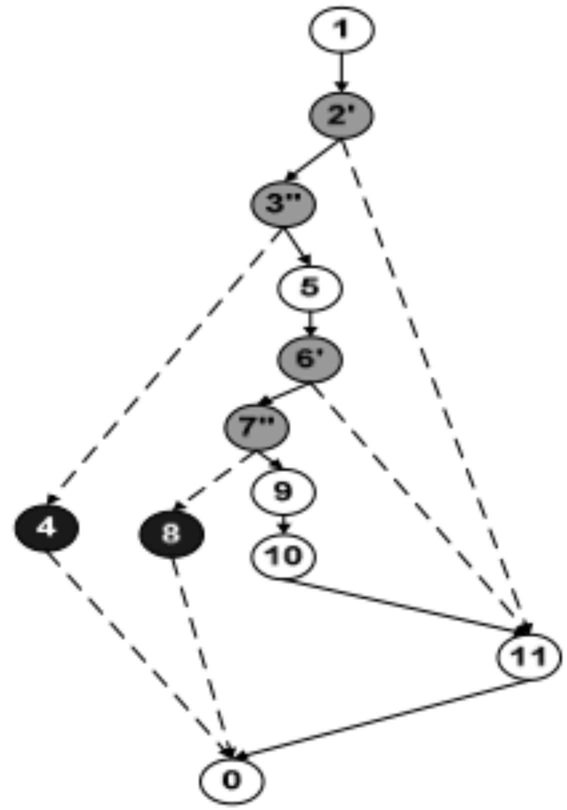
```
  public boolean
  sub3(int key)
  {
1 : Entry e = this.header;
2’: assume(e != null);
3”: assume !(e.value==key);
4 :

5 : e = e.next;
6’: assume (e != null);
7”: assume !(e.value==key);
8 :

9 : e = e.next;

10: assume(e == null);

11: return false;
0 :}
```



Only the definition in line 9 reaches exit

Discussion

- Limited experiments done so far
- Substantial speedup in higher scopes (around 6, 7)
  - Two rounds of splitting
- Small speedup when the complexity of the specification is more than the code formula
  - The benefit will be reduced by the overhead of multiple checking
- Because sub-graphs are independent, they can be checked in parallel

ESC/Java

- Extended Static Checker for Java
  - Finds common programming errors (not a prover!)
  - Compile-time checker
    - Catches more errors than a typical type checker
    - Examples:
      - Null dereference, array out of bound, type cast error
    - Examples of concurrent problems:
      - Race conditions
      - Deadlocks
    - Can also check user-defined design decisions (pre/post conditions)
- Based on
  - Verification-condition generation
  - Automatic theorem proving
- Uses its own annotation language

ESC/Java features

- ESC/Java is modular
  - Operates on one procedure at a time
  - Advantage: scalability
  - Disadvantage: user-provided annotations

- Is more lightweight than a full verification tool
  - Annotations are smaller

- Has to make a trade-off between
  - Missed errors (unsoundness)
  - False alarms (incompleteness)
  - Annotation overhead
  - performance

Running ESC/Java – example

```
class Bag {
    int size;
    int[] elements;   // valid:  elements[0..size-1]

    Bag(int[] input) {
        size = input.length;
        elements = new int[size];
        System.arraycopy(input, 0, elements, 0, size);
    }

    int extractMin() {
        int min = Integer.MAX_VALUE;
        int minIndex = 0;
        for (int i = 1; i <= size; i++) {
            if (elements[i] < min) {
                min = elements[i];
                minIndex = i;
            }
        }
        size--;
        elements[minIndex] = elements[size];
        return min;
    }
}
```

Running ESC/Java – example

```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
Bag.java:15: Warning: Possible null dereference (Null)
      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

Running ESC/Java – example

```
1:  class Bag {
2:        int size;
3:        int[] elements;    // valid:  elements[0..size-1]
4:
5:        Bag(int[] input) {
6:              size = input.length;
7:              elements = new int[size];
8:              System.arraycopy(input, 0, elements, 0, size);
9:        }
10:
11:       int extractMin() {
12:             int min = Integer.MAX_VALUE;
13:             int minIndex = 0;
14:             for (int i = 1; i <= size; i++) {
15:                   if (elements[i] < min) {
16:                         min = elements[i];
17:                         minIndex = i;
18:                   }
19:             }
20:             size--;
21:             elements[minIndex] = elements[size];
22:             return min;
23:       }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
Bag.java:15: Warning: Possible null dereference (Null)
      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

1 needs a pre-condition for constructor (or fixing the code)

```
4a:        //@ requires input != null
```

Running ESC/Java – example

```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
Bag.java:15: Warning: Possible null dereference (Null)
      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

```
3':        /*@non_null*/ int[] elements;
```

2 and 4 are there because elements is not private
– making it private doesn't remove warnings
– ESC can't check all methods to ensure elements is not assigned null
– (it's modular)

Running ESC/Java – example

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```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
Bag.java:15: Warning: Possible null dereference (Null)
      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

3 is because other code might mutate size

```
2a:   //@ invariant 0 <= size  && size <= elements.length
```

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Running ESC/Java – example

```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
Bag.java:15: Warning: Possible null dereference (Null)
      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

Even with the invariant,
it complains about index too large (line 15)

```
14:     for (int i = 1; i <= size; i++) {
to:
14':    for (int i = 0; i < size; i++) {
```

Running ESC/Java – example

```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:6: Warning: Possible null dereference (Null)
    size = input.length;
                ^
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      if (elements[i] < min) {
                  ^
Bag.java:15: Warning: Array index possibly too large (...
      if (elements[i] < min) {
                  ^
Bag.java:21: Warning: Possible null dereference (Null)
    elements[minIndex] = elements[size];
                                 ^
Bag.java:21: Warning: Possible negative array index (...
    elements[minIndex] = elements[size];
                                 ^
```

Last warning: procedure can be called when bag is empty (size = 0)

```
20a:        if (size >= 0) {
21:             elements[minIndex] = elements[size];
21a:        }
```

Running ESC/Java – example – 2nd run

```
1:  class Bag {
2:      int size;
3:      int[] elements;   // valid:  elements[0..size-1]
4:
5:      Bag(int[] input) {
6:          size = input.length;
7:          elements = new int[size];
8:          System.arraycopy(input, 0, elements, 0, size);
9:      }
10:
11:     int extractMin() {
12:         int min = Integer.MAX_VALUE;
13:         int minIndex = 0;
14:         for (int i = 1; i <= size; i++) {
15:             if (elements[i] < min) {
16:                 min = elements[i];
17:                 minIndex = i;
18:             }
19:         }
20:         size--;
21:         elements[minIndex] = elements[size];
22:         return min;
23:     }
24: }
```

```
Bag.java:26: Warning: Possible violation of object in-
variant
  }
  ^
Associated declaration is "Bag.java", line 3, col 6:
  //@ invariant 0 <= size  &&  size <= elements.length
    ^
Possibly relevant items from the counterexample context:
  brokenObj == this
(brokenObj* refers to the object for which the invariant
is broken.)
```

```
19a:        if (size > 0) {
 20:            size--;
 21:            elements[minIndex] = elements[size];
21a:        }
```

Line 26 is the old line 20.
Size may become negative

Running ESC/Java – example – 3rd run

```
class Bag {
    int size;
    int[] elements;   // valid:  elements[0..size-1]

    Bag(int[] input) {
        size = input.length;
        elements = new int[size];
        System.arraycopy(input, 0, elements, 0, size);
    }

    int extractMin() {
        int min = Integer.MAX_VALUE;
        int minIndex = 0;
        for (int i = 1; i <= size; i++) {
            if (elements[i] < min) {
                min = elements[i];
                minIndex = i;
            }
        }
        size--;
        elements[minIndex] = elements[size];
        return min;
    }
}
```

After all the fixes, no more warnings are reported.

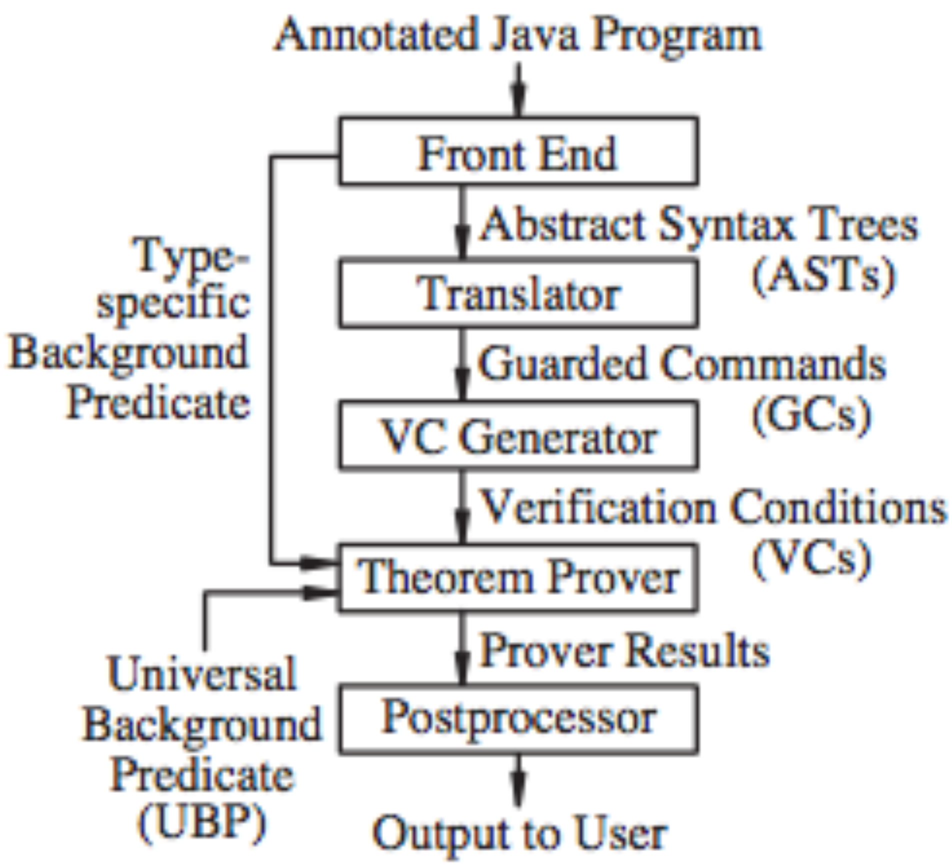
What did we learn

- Warnings resulted in
  - One pre-condition (inputs != null)
  - Two rep invariants (on size and elements)
  - Two bug fixes (wrong index range, missing case of empty bag)

- Using pre-conditions:
  - When checking a procedure foo, assumes that its pre-conditions hold
  - When encountering a call to foo, checks whether the pre-conditions hold or not

- Using object invariants (rep invariants):
  - Assumes that they hold in the pre-state
  - Checks whether they hold in the post-state or not

Architecture



Front-end

- Generates abstract syntax tree (AST)
- Generates type-specific background predicate
  - A formula in first-order logic
  - Generated for every class whose routines are to be checked
  - Encodes information about types and fields that routines use
  - Example: for a final class T
    - All S :: S <:T => S = T

Translation

- Generates Dijkstra's guarded commands (GC)
- Insert commands of the form **assert E** (E is a boolean expression)
- Ideal translation of a procedure R is to get a guarded command G s.t.
  - If there is a way that R starts from a state satisfying its precondition and behave erroneously (violate post conditions), G has at least one execution that starts in a state satisfying the precondition and then violates some assertion
  - If there is no way that R can start from a state satisfying its precondition and then behave erroneously, then G has no execution that starts in a state satisfying the precondition and then violates some assertion

- ESC Translation is neither sound nor complete
  - Neither of the above conditions holds

Translation

- Sources of inaccuracy:
  - Modularity
    - replacing calls with specs (usually under-specifications). We may report a bug that is not feasible in the code
    - Especially for ESC/Java, the specs are lightweight, supposed to encode only as much as needed for analysis
  - Overflow
    - We ignore arithmetic overflows. We may miss errors
  - Loops
    - unroll them (misses errors that need more iterations)
    - asking for loop invariants is unrealistic for practical code
    - default is one unrolling, but user can provide more

VC generation

- Generates verification conditions for each guarded command G
  - Is a predicate in first-order logic that holds for exactly those program states from which no execution of the command G goes wrong.
  - Computation similar to computing weakest pre-conditions + optimizations to avoid exponential blow-up

- An execution of a guarded command is said to “go wrong” if control reaches a subcommand of the form assert E when E is false

Thorem proving

- Uses Simplify
- Solves $\mathrm{UBP} \wedge \mathrm{BP}_T \Rightarrow \mathrm{VC}_R$
  - UBP: universal background predicate
  - BP: type-specific background predicate
  - VC_R: verification condition for procedure R

- Universal background predicate
  - Encodes facts about the semantics of Java
  - E.g. that the subtype relation is reflexive, anti-symmetric, and transitive

Post-processing

- Takes the theorem prover's output and generates warnings when proofs fail

- Simplify allows for
  - Labeled constraints
  - Can track back the source context corresponding to each constraint

- Since the formulas are in FOL (undecidable), the runtime of simplify is limited by some threshold
  - It might report something as a bug that could've been proved in longer time
  - (more false warnings)

Annotation language

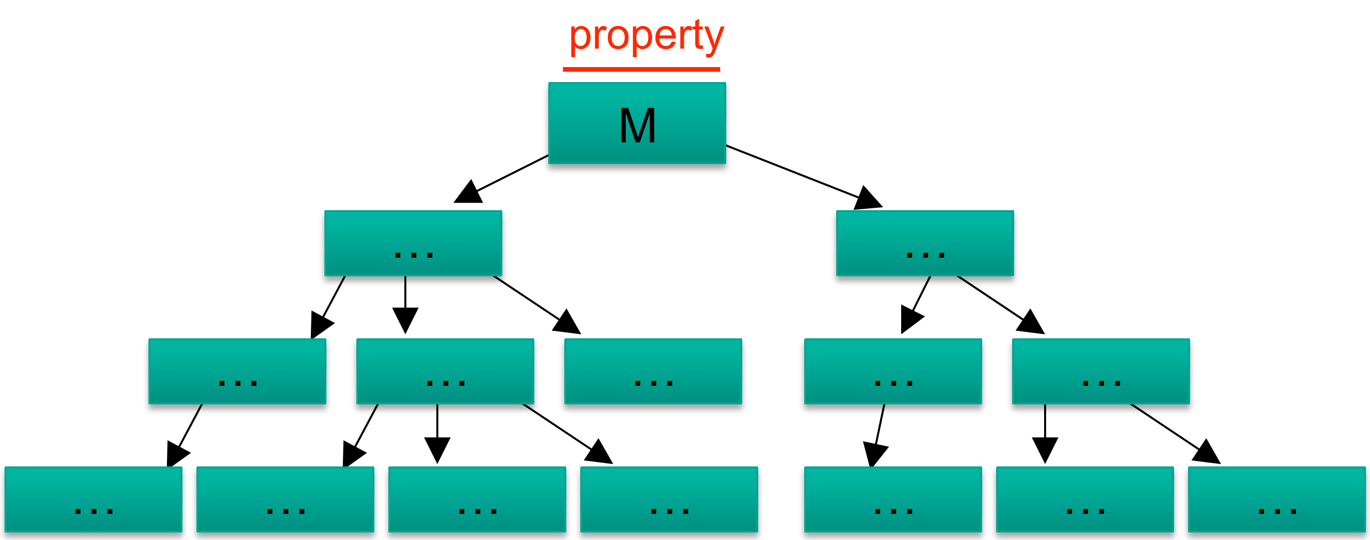
- Similar to JML, but small differences
  - JML is intended for full specification of programs
  - ESC/Java is intended for lightweight specifications
  - So small syntactic and semantic differences

- Cost:
  - Mostly small annotations (argument non-null, etc.)
  - 40-100 annotations per 1000 LOC (4-10%)
  - In the experiments, they were inserted interactively:
    - First annotated based on a rough understanding of code
    - Then ESC/Java ran, then more annotations added
  - Expensive on users
  - Prohibitively costly when running ESC on existing codebase

Program verification

- Construct a logical formula whose solutions are executions of the code that violate the property (*f*)
- Now solve (*f*)

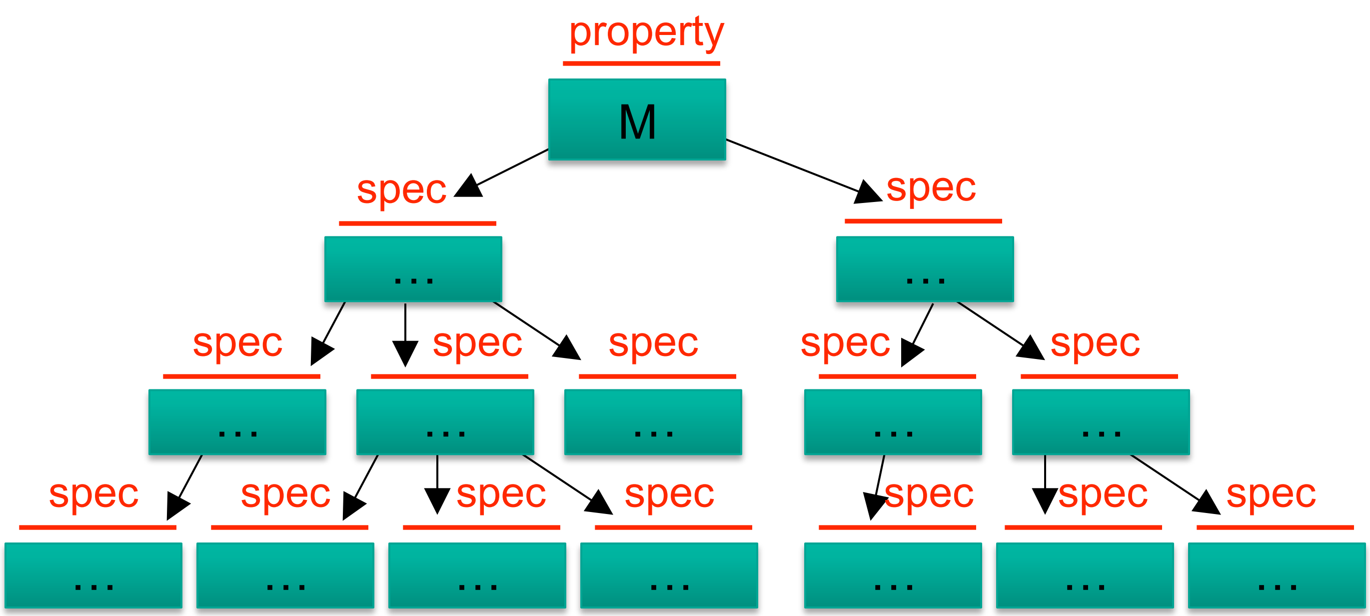
Either translate the code precisely, or ..



Modular analysis

- Replace a procedure with its specification
- Makes the technique better scalable
- But, is very costly for the user

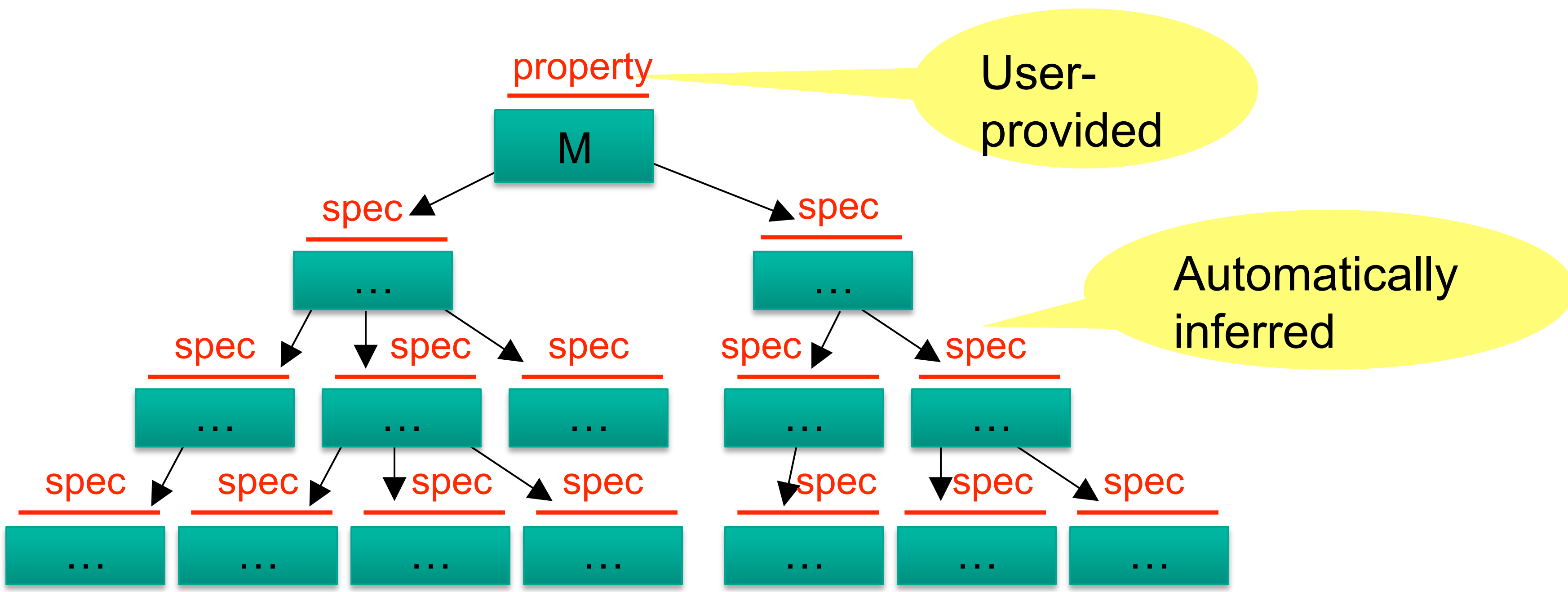
Ask for user-provided annotations, or..



Specification inference

- User provides only the top-level property
- This substantially reduces the human cost

Infer intermediate annotations automatically



ESC/Java annotations

- Simplest annotation-based analysis
  - Type checker is a limited program analysis tool
  - It is modular and requires type annotations from users
- ESC/Java is like an advanced type checker
  - Checks for null-dereference, array bounds, etc.
  - So it doesn't need extensive annotations like full verification
  - Still the amount of annotations can be up to 10% of the code size
- Houdini:
  - Generates intermediate annotations automatically

Houdini – Annotation assistant for ESC/Java

Input: An unannotated program P
Output: ESC/Java warnings for an annotated version of P
Algorithm:
generate set of candidate annotations and insert into P;
repeat
  invoke ESC/Java to check P;
  remove any refuted candidate annotations from P;
until quiescence;
invoke ESC/Java to identify possible defects in P;

- Generating candidates is done by looking at program text
- It uses heuristics about what annotations might be useful
- Example:
  - all preconditions of the form argument != null

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- To identify incorrect annotations:
  - Invoke ESC/Java
  - Ignore warnings about runtime errors (e.g. null dereference)
  - If there is a warning about an annotation not true at some program point (e.g. a method's precondition doesn't hold at a call site), then remove that annotation from the candidate set
  - Removing one annotation may make others invalid, so repeat until fixpoint

Algorithm properties

- Remaining annotations are a subset of the initial candidate set
- Are guaranteed to be valid as much as ESC can tell
- They represent a maximal valid subset of the candidate set

- After the check-refute cycle, Houdini runs ESC/Java again
- This identifies potential run-time errors in the new annotated program
- These warnings are output to the user

Candidate annotations

- Ideally, the initial set must contains “all” possible annotations
- But, the set cannot be too big because of performance
- Following heuristics are based on experiments
- For a field f, we generate the following invariants:

Type of `f`	Candidate invariants for `f`
integral type	`//@ invariant f` *cmp expr*`;`
reference type	`//@ invariant f != null;`

array type	`//@ invariant f != null;`
	`//@ invariant \nonnullelements(f);`
	`//@ invariant (\forall int i; 0 <= i && i <` *expr*
	`==> f[i] != null);`
	`//@ invariant f.length` *cmp expr*`;`

boolean	`//@ invariant f == false;` `//@ invariant f == true;`

Generated invariants

- Integral invariants
  - Mainly to check array index out of bound
  - Comparison operators: <, <=, ==, !=, >=, >
  - Comparison expression:
    - an integer field declared earlier in the same class
    - Or an interesting constant: -1, 0, 1, array dimensions (new int [4])
  - Contradicting invariants are no problem (x < 0 and x >= 0)
    - One of them gets refuted very fast
- Reference invariants
  - To check pointer != null
  - Array pointers non-null
  - Array elements non-null
  - Array elements up to expr (a field or a constant) non-null
    - Useful in checking stack implemented by array

Other annotations

- Candidate pre-conditions
  - Comparison of two arguments
  - Relating an argument to a field declared in the same class
  - Also **//@requires false**
    - Any unrefuted precondition of this form shows the procedure is never called
    - To identify dead code
- Candidate post-conditions
  - Relate the \result to an argument
  - Relate the \result to a field
  - Also //@ensures \fresh(\result)
    - That result is a newly allocated object

Experimental results

- Houdini is applied to a few programs of various sizes, up to 36kLOC
- It reduces the number of warnings of ESC/Java substantially

Type of annotation	Preconditions guessed	Preconditions %valid	Postconditions guessed	Postconditions %valid	Invariants guessed	Invariants %valid	Total guessed	Total %valid
`f == ` *expr*	2130	18	985	18	435	14	3550	17
`f != ` *expr*	2130	35	985	35	435	38	3550	35
`f < ` *expr*	2130	26	985	27	435	24	3550	26
`f <= ` *expr*	2130	31	985	32	435	36	3550	33
`f >= ` *expr*	2130	25	985	21	435	19	3550	32
`f > ` *expr*	2130	31	985	36	435	35	3550	23
`f != null`	509	92	229	79	983	72	1721	79
`\nonnullelems(f)`	54	81	21	62	36	64	111	72
`(\forall ...)`	841	27	260	37	125	59	1226	32
`f == false`	47	36	51	25	39	10	137	20
`f == true`	47	28	51	24	39	8	137	25
`\fresh(\result)`	0	0	229	30	0	0	229	30
`false`	780	17	0	0	0	0	780	17
exact type	37	19	11	36	14	57	62	31
Total	15095	30	6762	30	3846	40	25703	31