Characterizing Trees

A directed tree is a directed graph in which one of the nodes – called root – is distinguished from the others, and has the following property: for any other node v, there is exactly one directed path from root to v. A directed tree can be represented in Alloy by its set of nodes and edges plus some additional constraints.

Task 1: Express these constraints using the following template, and illustrate a few trees using the Alloy visualizer.

```alloy
sig Node { e : set Node }
fact { ... } // replace this with your constraints
pred show() {}
run show for 6
```

Task 2: To make sure that your formulation is correct, check that the number of edges in your tree equals the number of nodes minus one. Express this as an assertion and use the Alloy Analyzer to verify it.

Solution

```alloy
sig Node {
    e : set Node
}

one sig Root in Node {};

fact {
    Node in Root.*e  // connectivity
    no e.Root        // implies acyclicity
    all n : Node | lone e.n // at most one parent
}

pred show() {}
run show for 10

assert cardinality {
    #e = #Node - 1
}

check cardinality for 8
```