JKelloy: A Proof Assistant for Relational Specifications of Java Programs

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Bringing together

Java + ALLOY

- Classes
- Fields
- Linked Data Structures
- Program states

- Sets
- Relations
- First-Order Relational Logic
- No states

Relational Specifications are Concise

```
class Tree {
  Tree left, right;
  Data data;
}
```

$$\rightsquigarrow \quad \begin{aligned} \mathit{Tree}, \mathit{Data} &\subseteq \mathit{Object} \\ \mathit{left}, \mathit{right} &\subseteq \mathit{Tree} \times \mathit{Tree} \cup \mathit{Null} \\ \mathit{data} &\subseteq \mathit{Tree} \times \mathit{Data} \cup \mathit{Null} \end{aligned}$$

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- All instances of `Tree` are acyclic:

$$\forall t : \mathit{Tree} \mid t \notin t\,\texttt{.}\,\hat{}\,(\mathit{left}\ \texttt{+}\ \mathit{right})$$

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- A tree t is a subtree of the tree `root`:

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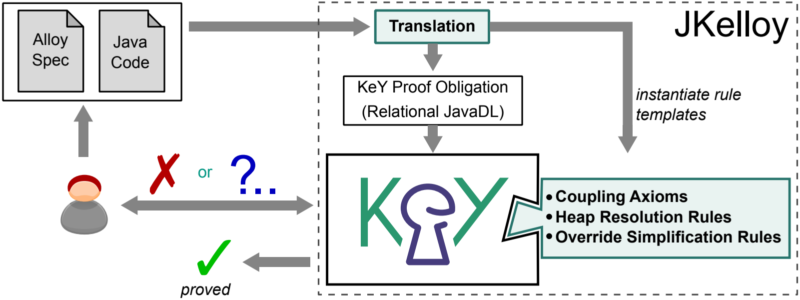
- A tree t is a subtree of the tree `root`:

$$t \in root\,.\,{*}(left\ +\ right)$$

- The data d occurs at most once in the tree `root`:

$$\#((root\,.\,{*}(left\ +\ right))\ \&\ (data\,.\,d)) \leq 1$$

JKelloy – *Framework*



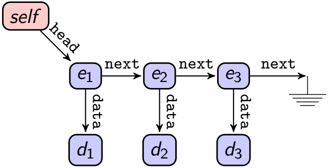
- Alloy as specification language
- KeY for symbolic execution
- Heap resolution rules: Translate Java state to Alloy
- Override resolution rules: Simplify relational expressions

Prepend Example

```
class List {
 Entry head;

 void prepend(Object d) {
   Entry oldHead = head;
   head = new Entry();
   head.next = oldHead;
   head.data = d;
 }
}

class Entry {
 Data data;
 Entry next;
}
```



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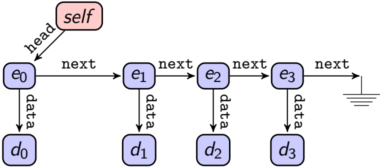

Construction of Proof Obligations

$$pre \rightarrow [\texttt{prepend(d);}]post$$

symbolic execution

$$pre \wedge path_1 \rightarrow \{\mathcal{U}_1\}post \quad \cdots \quad pre \wedge path_n \rightarrow \{\mathcal{U}_n\}post$$

Construction of Proof Obligations

$$pre \rightarrow \{h_s := heap\}[\texttt{prepend(d);}]\{h_e := heap\}post$$

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$\downarrow$

$head = newEntry() \rightsquigarrow \qquad create(h_s, e)$

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$head = newEntry() \rightsquigarrow$

$$h_s$$
$$\downarrow$$
$$create(h_s, e)$$
$$\downarrow$$
$$store(create(h_s, e), self, \texttt{head}, e)$$

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$head.next = oldHead \rightsquigarrow$

$$store(store(create(h_s, e), self, \texttt{head}, e), e, \texttt{next}, h_s[self.\texttt{head}])$$

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$head.next = oldHead \rightsquigarrow$

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$head.data = d \rightsquigarrow$

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Coupling Alloy and Java States

```
class List {
 Entry head;
 void prepend(Data d) {...}
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class Entry {
 Data data;
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$$\begin{aligned} \mathit{List}, \mathit{Entry}, \mathit{Data} &\subseteq \mathit{Object} \\ \mathit{head} &\subseteq \mathit{List} \times \mathit{Entry} \cup \mathit{Null} \\ \mathit{data} &\subseteq \mathit{Entry} \times \mathit{Data} \cup \mathit{Null} \\ \mathit{next} &\subseteq \mathit{Entry} \times \mathit{Entry} \cup \mathit{Null} \end{aligned}$$

Coupling Axioms in JavaDL (KeY 's logic)

$$\mathit{Entry}_{rel}(h) := \{o \mid h[o.\langle\texttt{created}\rangle] \ \wedge \ o \sqsubseteq \mathit{Entry} \ \wedge \ o \neq \texttt{null}\}$$

$$\mathit{data}_{rel}(h) := \{(o_1, o_2) \mid o_1 \in \mathit{Entry}_{rel}(h) \ \wedge \ (o_2 = \texttt{null} \vee o_2 \in \mathit{Data}_{rel}(h)) \ \wedge \ o_2 = h[o_1.\texttt{data}]\}$$

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Evaluating an Alloy relation in a Java heap context

$$a\,.\,f(store(h_i, o, g, v)) \stackrel{?}{=} b$$

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$(b = \texttt{null} \vee b \in T(store(h_i, o, g, v))) \wedge$
$b = store(h_i, o, g, v)[a.\texttt{f}]$

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Select/Store Axioms

$$store(h_i, o, g, v)[a.\mathtt{f}] = \begin{cases} v & \text{if } o = a \wedge \mathtt{g} = \mathtt{f} \wedge \mathtt{g} \neq \mathtt{created} \\ h_i[a.\mathtt{f}] & \text{else} \end{cases}$$

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→ **Relational Heap Resolution**

Heap Resolution Rules – *Example*

- Postcondition of `prepend(d)` (in Alloy)

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$$\{self\} \,.\, head_{rel}(h_e) \,.\, next_{rel}(h_e)^* \,.\, data_{rel}(h_e)$$
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$$head_{rel}(store(h_4, e, \mathtt{data}, d)) \rightsquigarrow$$

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$$head_{rel}(store(h_2, self, \mathtt{head}, e)) \rightsquigarrow$$

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- Relational heap resolution

$$head_{rel}(store(h_2, self, \mathtt{head}, e)) \rightsquigarrow head_{rel}(create(h_s, e)) \oplus \{self\} \times \{e\}$$

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- Relational heap resolution

$$\begin{aligned}head_{rel}(h_e) &\overset{\cdots}{\rightsquigarrow} head_{rel}(h_s) \oplus \{self\} \times \{e\}\\ next_{rel}(h_e) &\overset{\cdots}{\rightsquigarrow} next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s)\\ data_{rel}(h_e) &\overset{\cdots}{\rightsquigarrow} data_{rel}(h_s) \oplus \{e\} \times \{d\}\end{aligned}$$

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- Postcondition after Heap Resolution

$$\begin{aligned}&\{self\} \,.\, (head_{rel}(h_s) \oplus \{self\} \times \{e\})\\ &.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^{*} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\})\\ =&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{*} \,.\, data_{rel}(h_s) \cup \{d\}\end{aligned}$$

Override Resolution Rules: Example 1

$\mathbf{R_1}$**:** $\{a\} . (R \oplus \{a\} \times \{b\}) \rightsquigarrow \{b\}$

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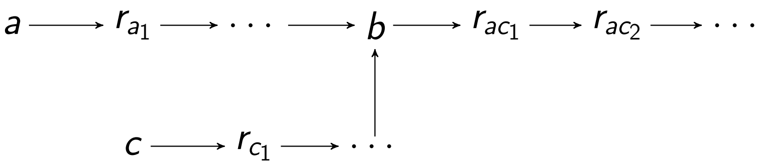
$\mathbf{R}_1$**:** $\{a\} . (R \oplus \{a\} \times \{b\}) \rightsquigarrow \{b\}$

Override Resolution Rules: Example 2

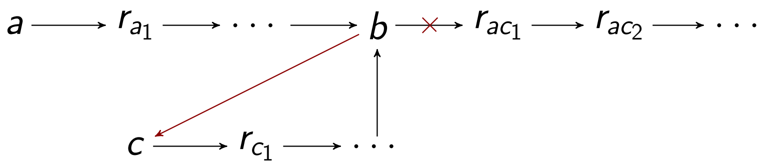


$$\mathbf{R_3:}\ \{\mathbf{a}\} \,.\, (\mathbf{R} \oplus \{\mathbf{b}\} \times \{\mathbf{c}\})^+ \rightsquigarrow$$

$$\begin{array}{ll} \text{if } b \in \{c\}.R^+ \vee b = c & \text{then } (\{\mathbf{a}\} \,.\, \mathbf{R}^+ \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^+) \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^+ \\ & \text{else } (\{\mathbf{a}\} \,.\, \mathbf{R}^+ \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^+) \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^+ \end{array}$$

assuming $b \in \{a\} \,.\, R^+$, $parFun(R)$ and $acyc(R)$

Override Resolution Rules: Example 2



$\mathbf{R_3}$: $\{\mathbf{a}\} \,.\, (\mathbf{R} \oplus \{\mathbf{b}\} \times \{\mathbf{c}\})^+ \leadsto$

if $b \in \{c\}.R^+ \vee b = c$ then $(\{\mathbf{a}\} \,.\, \mathbf{R}^+ \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^+) \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^+$

else $(\{\mathbf{a}\} \,.\, \mathbf{R}^+ \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^+) \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^+$

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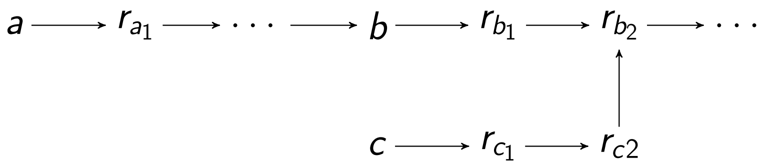
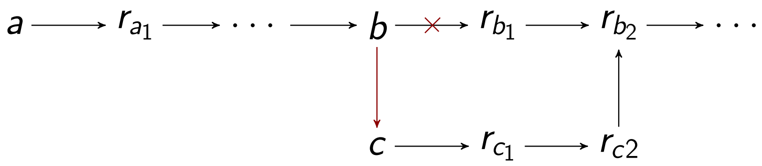


$\mathbf{R}_3$**:** $\{\mathbf{a}\}\,.\,(\mathbf{R} \oplus \{\mathbf{b}\} \times \{\mathbf{c}\})^+ \rightsquigarrow$

$$\text{if } b \in \{c\}.R^+ \vee b = c \quad \text{then } (\{\mathbf{a}\}\,.\,\mathbf{R}^+ \cup \{\mathbf{c}\} \cup \{\mathbf{c}\}\,.\,\mathbf{R}^+) \setminus \{\mathbf{b}\}\,.\,\mathbf{R}^+$$
$$\text{else } (\{\mathbf{a}\}\,.\,\mathbf{R}^+ \setminus \{\mathbf{b}\}\,.\,\mathbf{R}^+) \cup \{\mathbf{c}\} \cup \{\mathbf{c}\}\,.\,\mathbf{R}^+$$

assuming $b \in \{a\}\,.\,R^+$, *parFun*(R) and *acyc*(R)

Override Resolution Rules: Example 2



$$\mathbf{R_3}\mathbf{:}\ \{\mathbf{a}\} \,.\, (\mathbf{R} \oplus \{\mathbf{b}\} \times \{\mathbf{c}\})^{+} \rightsquigarrow$$

$$\text{if } b \in \{c\}.R^{+} \lor b = c \quad \text{then } (\{\mathbf{a}\} \,.\, \mathbf{R}^{+} \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^{+}) \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^{+}$$

$$\text{else } (\{\mathbf{a}\} \,.\, \mathbf{R}^{+} \setminus \{\mathbf{b}\} \,.\, \mathbf{R}^{+}) \cup \{\mathbf{c}\} \cup \{\mathbf{c}\} \,.\, \mathbf{R}^{+}$$

assuming $b \in \{a\} \,.\, R^{+}$, $parFun(R)$ and $acyc(R)$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{self\} \,.\, (head_{rel}(h_s) \oplus \{self\} \times \{e\}) \,.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^* \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$

$$= \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^* \,.\, data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{self\} . (head_{rel}(h_s) \oplus \{self\} \times \{e\}) \\
&. (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^* . (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
= &\{self\} . head_{rel}(h_s) . next_{rel}(h_s)^* . data_{rel}(h_s) \cup \{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{self\} \,.\, (head_{rel}(h_s) \oplus \{self\} \times \{e\})$$
$$.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^* \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^* \,.\, data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{self\} \,.\, (head_{rel}(h_s) \oplus \{self\} \times \{e\})$$
$$.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^* \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^* \,.\, data_{rel}(h_s) \cup \{d\}$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{self\} \,.\, (head_{rel}(h_s) \oplus \{self\} \times \{e\})$$
$$.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^* \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^* \,.\, data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{e\} . (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^* . (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$=\{self\} . head_{rel}(h_s) . next_{rel}(h_s)^* . data_{rel}(h_s) \cup \{d\}$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{e\} \mathbin{.} (next_{rel}(h_s) \oplus \{e\} \times \{self\} \mathbin{.} head_{rel}(h_s))^* \mathbin{.} (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= \{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} next_{rel}(h_s)^* \mathbin{.} data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$(\{e\} \cup \{e\} . (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^{+}) . (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= (\{self\} . head_{rel}(h_s) \cup \{self\} . head_{rel}(h_s) . next_{rel}(h_s)^{+}) . data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$(\{e\} \cup \{e\} . (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^{+}) . (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= (\{self\} . head_{rel}(h_s) \cup \{self\} . head_{rel}(h_s) . next_{rel}(h_s)^{+}) . data_{rel}(h_s) \cup \{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{e\} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \cup \\
&\{e\} \,.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^{+} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
=&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$\{e\} . (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \cup$$
$$\{e\} . (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^+ . (data_{rel}(h_s) \oplus \{e\} \times \{d\})$$
$$= \{self\} . head_{rel}(h_s) . data_{rel}(h_s) \cup$$
$$\{self\} . head_{rel}(h_s) . next_{rel}(h_s)^+ . data_{rel}(h_s) \cup$$
$$\{d\}$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
& \{d\} \cup \\
& \{e\} . (next_{rel}(h_s) \oplus \{e\} \times \{self\} . head_{rel}(h_s))^{+} . (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
= & \{self\} . head_{rel}(h_s) . data_{rel}(h_s) \cup \\
& \{self\} . head_{rel}(h_s) . next_{rel}(h_s)^{+} . data_{rel}(h_s) \cup \\
& \{d\}
\end{aligned}
$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{d\} \cup \\
&\{e\} \,.\, (next_{rel}(h_s) \oplus \{e\} \times \{self\} \,.\, head_{rel}(h_s))^{+} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
=&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
& \{d\} \cup \\
& (\{self\} \,.\, head_{rel}(h_s) \cup \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+}) \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
= & \{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
& \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
& \{d\}
\end{aligned}
$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{d\} \cup \\
&(\{self\} \,.\, head_{rel}(h_s) \cup \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+}) \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
=&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{d\} \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
=&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
& \{d\} \cup \\
& \{self\} \,.\, head_{rel}(h_s) \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \cup \\
& \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
= & \{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
& \{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^{+} \,.\, data_{rel}(h_s) \cup \\
& \{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{d\} \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^+ \,.\, (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
=&\{self\} \,.\, head_{rel}(h_s) \,.\, data_{rel}(h_s) \cup \\
&\{self\} \,.\, head_{rel}(h_s) \,.\, next_{rel}(h_s)^+ \,.\, data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
& \{d\} \cup \\
& \{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} data_{rel}(h_s) \cup \\
& \{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} next_{rel}(h_s)^{+} \mathbin{.} (data_{rel}(h_s) \oplus \{e\} \times \{d\}) \\
= & \{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} data_{rel}(h_s) \cup \\
& \{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} next_{rel}(h_s)^{+} \mathbin{.} data_{rel}(h_s) \cup \\
& \{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{d\} \cup \\
&\{self\} . head_{rel}(h_s) . data_{rel}(h_s) \cup \\
&\{self\} . head_{rel}(h_s) . next_{rel}(h_s)^+ . data_{rel}(h_s) \\
= &\{self\} . head_{rel}(h_s) . data_{rel}(h_s) \cup \\
&\{self\} . head_{rel}(h_s) . next_{rel}(h_s)^+ . data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of `prepend` using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

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\begin{aligned}
&\{d\} \cup \\
&\{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} data_{rel}(h_s) \cup \\
&\{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} next_{rel}(h_s)^{+} \mathbin{.} data_{rel}(h_s) \\
=&\{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} data_{rel}(h_s) \cup \\
&\{self\} \mathbin{.} head_{rel}(h_s) \mathbin{.} next_{rel}(h_s)^{+} \mathbin{.} data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

Proof of prepend using Override Resolution Rules

Postcondition of 'prepend(d)' after heap resolution

$$
\begin{aligned}
&\{self\}\,.\,head_{rel}(h_s)\,.\,data_{rel}(h_s) \cup \\
&\{self\}\,.\,head_{rel}(h_s)\,.\,next_{rel}(h_s)^{+}\,.\,data_{rel}(h_s) \cup \\
&\{d\} \\
=&\{self\}\,.\,head_{rel}(h_s)\,.\,data_{rel}(h_s) \cup \\
&\{self\}\,.\,head_{rel}(h_s)\,.\,next_{rel}(h_s)^{+}\,.\,data_{rel}(h_s) \cup \\
&\{d\}
\end{aligned}
$$

More Override Resolution Rules

Further rules to reason about the assumptions and conditions of the main rules

- Partial funtionality preservation, e.g.
  $\mathbf{R}_8$: $\vdash \mathit{parFun}(\mathbf{R_1}) \wedge \mathit{parFun}(\mathbf{R_2}) \rightarrow \mathit{parFun}(\mathbf{R_1} \oplus \mathbf{R_2})$
- Acyclicity preservation, e.g.
  $\mathbf{R}_9$: $\vdash \mathit{acyc}(\mathbf{R}) \wedge \mathbf{R} \,.\, \{\mathbf{a}\} = \emptyset \wedge \mathbf{a} \neq \mathbf{b} \rightarrow \mathit{acyc}(\mathbf{R} \oplus \{\mathbf{a}\} \times \{\mathbf{b}\})$
- Reachability lemmas, e.g.
  $\mathbf{R}_{12}$: $S_2 \in S_1 \,.\, R^+ \rightsquigarrow \mathbf{false}$ assuming $S_1 \,.\, R = \emptyset$
- See TR for the general fragment supported by the override simplification rules

Evaluation

- `List.prepend` fully automatic, 1546 RuleApps, 5.4 sec
- `List.append`, 28 interactive, 2850 RuleApps
- `Graph.remove`, 1201 interactive, 6973 RuleApps

Potential to optimize proof strategy

Conclusion

- Deductive verification of Java programs, concise Alloy specs
- Problem we addressed: How does a data structure change as a whole, when modifying a Java field
- Concise proofs of linked data structure without induction
- Coupling Axioms, 2 sets of simplification rules
- Evaluation: promissing approach; early development stage